Exam paper solu

(1) If the students calculate mass transfer coefficients by any of the three film theory taught in class, then it is ok

e.g. typical interfacial thickness ~ 100 μm = 0.01 cm

$$k = \frac{D}{l} = \frac{2.5 \times 10^{-5}}{0.01} \ cm/s = 2.5 \times 10^{-3} \ cm/s$$

or $k = \sqrt{\frac{D}{\tau}}$ when $\tau = 3\,s$ (1–5 seconds is ok)

$$= \sqrt{\frac{2.5 \times 10^{-5}}{3} \frac{cm^2}{s^2}} = 2.88 \times 10^{-3} \ cm/s$$

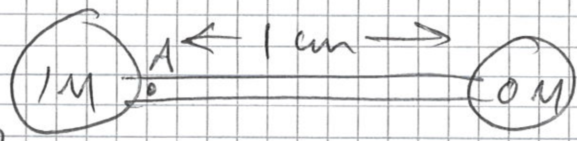
(2) Generally for membranes, when ~~the~~ we have carrier liquid (facilitating sites), we can have facilitated transport

Initially, the flux will increase as a function of concentration

However at high solute concentration, we will reach carrier saturation, resulting in constant flux.

3

(a) To obtain velocity, we need flux and concentration



Diffusion Flux $= -D\nabla C = 10^{-5}\frac{cm^2}{s}\times\frac{1\ mole}{1000\ cm^3}\times\frac{1}{1\ cm}$

$= 10^{-8}\frac{mole}{cm^2\cdot s}$

Velocity $= \frac{\text{Flux}}{\text{Concentr.}} = 10^{-8}\frac{mol}{cm^2\cdot s}\times\frac{1000\ cm^3}{1\ mole}$

$= 10^{-5}\ cm/s$

(b) Velocity imparted by drift (electric field) can be obtained by finding flux contribution

Flux contribution by e field $= -D z_i c_i \frac{F\nabla\Psi}{RT}$

$= -10^{-5}\frac{cm^2}{s}\times 1\times\frac{1\ \cancel{mole}}{1000\ cm^3}\times\frac{24.9\times10^3}{8.31\times300}\frac{\cancel{Joule}}{cm\ \cancel{Joule}}\ \cancel{mol}$

$= \frac{\cancel{10^{-8}}\ 10^{-10}\times 24.9\times10^3\ mole}{cm^2\cdot s\times 8.31\times3\times\cancel{10^2}}$

$= 10^{-10}\times10^{3}\frac{mole}{cm^2\cdot s}$

$= 10^{-7}\frac{mole}{cm^2\cdot s}$

Velocity from Drift $= 10^{-6}\ cm/s$

Total velocity $= 10^{-5}+10^{-6} = 1.1\times10^{-5}\ cm/s$

2

(4) ~~ex~~ 1D diffusion is 3x faster than 3D diffusion

$\Rightarrow D = 3 \times 1\ cm^2/s = 3\ cm^2/s$

(5) $D_1 = \infty$ because concentration do not change

$$D_2 = \frac{J}{\nabla C} = 10^{-8} \frac{\cancel{mole}}{cm^2/s} \times \frac{1000\ cm^3}{1\ \cancel{mole}} (1\ cm)$$

$$= \cancel{1000}\ 10^{-5} \frac{cm^2}{s}$$

$$\text{velocity} = \frac{J}{c_i} = \cancel{10^{-5}}\ 10^{-8} \frac{\cancel{mole}}{cm^2 \cdot s} \cdot \frac{1000\ cm^3}{\cancel{1\ mole}}$$

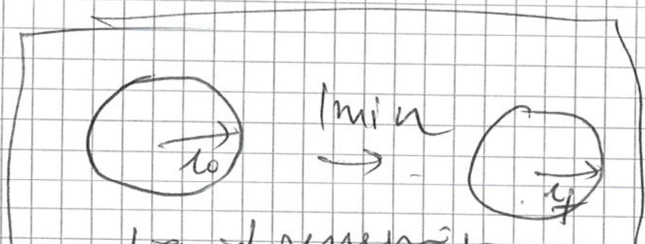
$$= 10^{-5}\ cm/s$$

This velocity is same at c_0 and c_i

because flux is same (steady-state)

and concentration is same.

(6)



Mass transfer on bubble

$$\frac{d}{dt}(c_1 V) = -k_1 A\, c_{1,sat}$$

$$\Rightarrow c_1 \frac{dV}{dt} = -k_1 c_{1sat} A$$

$$\Rightarrow \frac{dr}{dt} = -k_1 \frac{c_{1sat}}{c_1} \Rightarrow r = r_0 - k_1 \frac{c_{1sat}}{c_1} t$$

$$C_{1,sat} = 0.1\ mM = 0.1 \times 10^{-3} \frac{mol}{1000\ cm^3} = 10^{-7} \frac{mol}{cm^3}$$

C_1 is concentration inside the bubble at 1 bar

$$\Rightarrow C_1 = \frac{P_1}{RT} = \frac{10^5}{8.314 \times 300} \frac{mole}{m^3} = 40.1 \frac{mol}{m^3}$$

$$= 40.1 \times 10^{-6} \frac{mol}{cm^3}$$

$$= 4.01 \times 10^{-5} \frac{mol}{cm^3}$$

$$r = r_0 - k_1 \frac{C_{1,sat}}{C_1} t$$

$$\Rightarrow k_1 = \frac{(r_0 - r)}{C_{1,sat}} \cdot \frac{C_1}{t} = \frac{(1\ cm - 0.9\ cm)\ \cancel{4.01 \times 10^{-5} \frac{mol}{cm^3}}}{\cancel{10^{-7} \frac{mol}{cm^3}}\ \cancel{100\ s}}$$

$$= 0.1 \times 4.01\ cm/s$$

$$\Rightarrow k_1 = 0.04\ cm/s$$

(b) $\text{flux} = -k_1 A C_{1,sat}$

$$= 0.04 \frac{cm}{s} \times 4\pi \times 1\ cm^2 \times 0.1 \times 10^{-3} \frac{mole}{1000\ cm^3}$$

$$= 4 \times 4\pi \times 10^{-9} \frac{mol}{cm^2 \cdot s}$$

$$= 5.02 \times 10^{-8} \frac{mol}{cm^2 \cdot s}$$

(c) ~~$r = r_0 - k_1 \cdot \frac{C_{1,sat}}{C_1} t \Rightarrow r = 1\ cm - 0.04 \frac{cm}{s} \times \frac{0.1 \times 10^{-3}}{1000 \times 4.01 \times 10^{-5}}$~~

→ ~~x = 1 cm~~

(c) bubble shrinkage rate v is linear

so it will shrink by another 0.1 cm

to a size of 0.8 cm.

(7)

(a) Flux of evaporated water at $z=0$ will be ~~higher~~ equal compared to $z = L$. This is because total flux is constant. (boundary condition at $z=0$)

(b) velocity of water is changing because c is changing

$$\text{flux} = vc = \text{constant}$$

at $z=0$, concentration of water is highest

$\Rightarrow$ velocity is lowest.

At $z = L$, velocity is highest

(c) flux of water vapor at 0°C can be estimated by diffusive flux

$$= -D\nabla c = 10^{-2}\,\frac{cm^2}{s} \times \nabla c$$

(close to freezing point)

∇c can be obtained from ∇P

$$c = \frac{P}{RT} \qquad \nabla c = \frac{\nabla P}{RT}$$

$$\nabla c = \frac{(0.006 - 0.0001) \times 10^5}{8.314 \times 273}\,\frac{mol}{m^3} \times \frac{1}{10\,cm}$$

$$= \frac{59 \times 10^{-4} \times 10^5}{8.314 \times 273} \times \frac{1}{10^6}\,\frac{mol}{cm^3} \times \frac{1}{10\,cm} = 2.6 \times 10^{-7}\,\frac{mol}{cm^3} \times \frac{1}{10\,cm}$$

$\Rightarrow$ Flux $= 10^{-2} \frac{cm^2}{s} \times 2.6\times10^{-7} \frac{mol}{cm^3} \times \frac{1}{10\,cm}$

$= 2.6\times10^{-10} \frac{mol}{cm^2 \cdot s}$

Flux is constant & does not change.

~~(d) diffusive flux will be h~~

(8) ~~(a)~~ For $V_{LJ} = 0 \Rightarrow \left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6} = 0$

$\Rightarrow \left(\frac{\sigma}{r}\right)^{12} = \left(\frac{\sigma}{r}\right)^{6}$

$= \left(\frac{\sigma}{r}\right)^{6} = 1$

$\Rightarrow \sigma = r$

so, we need to calculate σ for Ar

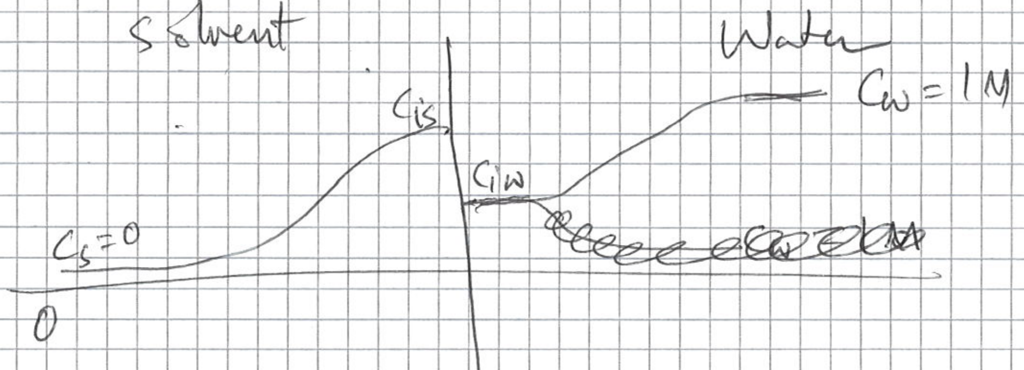
$\frac{\pi}{4}\sigma^2 = \frac{k_B T/P}{d}$

$\Rightarrow \frac{\pi}{4}\sigma^2 = \frac{(8.314/6.023\times10^{23}) \times 300}{10^5 \times 400\times10^{-9}}$

$= 1.03\times10^{-19}$

$\Rightarrow \sigma = 3.6\times10^{-10}\,m = 3.6\,\mathring{A}$

(9)



$$k_s = \sqrt{\frac{D_s}{\tau_s}} = \sqrt{\frac{1.25 \times 10^{-4}}{5} \frac{cm^2}{s^2}} = 5 \times 10^{-3} cm/s$$

$$k_w = \sqrt{\frac{D_w}{\tau_w}} = \sqrt{\frac{2.7 \times 10^{-5}}{3} \frac{cm^2}{s^2}} = 3 \times 10^{-3} cm/s$$

$$J = k_s (C_{is} - C_s) = k_w \left(\frac{C_w - C_{iw}}{\cancel{C_{iw} - C_w}} \right)$$

$$C_{is} = 10 C_{iw}$$

$$\Rightarrow k_s (10 C_{iw} - C_s) = k_w (C_w - C_{iw}) = J$$

$$\frac{10 J}{k_w} + \frac{J}{k_s} = \cancel{10 k_w} \left(10 C_w - \cancel{10 C_{iw}} + \cancel{10 C_{iw}} - C_s \right)$$

$$= 10 C_w - C_s$$

$$J = \frac{10 C_w - C_s}{\left(\frac{10}{k_w} + \frac{1}{k_s} \right)} = \frac{10 \ mol}{1000 \ cm^3} \frac{(cm/s)}{\left(\frac{10}{3 \times 10^{-3}} + \frac{1}{5 \times 10^{-3}} \right)}$$

$$= \frac{1}{100} \frac{mol}{cm^3} * \frac{15 \times 10^{-6}}{53 \times 10^{-3}} \ cm/s$$

$$= \frac{15}{53} \times 10^{-5} \ \frac{mol}{cm^2 \cdot s} = 2.83 \times 10^{-6} \frac{mol}{cm^2 \cdot s}$$

The flux is same in both phases. (steady state)

(b) $J = k_s(C_{is} - C_s)$

$$\Rightarrow C_{is} - C_s = J/k_s = \frac{2.83 \times 10^{-6} \text{ mol/cm}^2\text{s}}{5 \times 10^{-3} \text{ cm/s}}$$

$$= 0.56 \frac{\text{mol}}{\text{cm}^3}$$

$\Rightarrow C_{is} = 0.56 \frac{\text{mol}}{\text{cm}^3}$ $C_s = 0$

$\Rightarrow C_{iw} = 0.056 \frac{\text{mol}}{\text{cm}^3}$ $C_{is} = 10 C_{iw}$

(10) $k_0 = 0.01$ cm/s

$D = k_0 * l = 0.01 \frac{\text{cm}}{\text{s}} * 0.01 \text{ cm} = 10^{-4} \frac{\text{cm}^2}{\text{s}}$

$$\frac{\sqrt{kD}}{k^0} = \frac{\sqrt{10^4 * 10^{-4}}}{0.01} = 100$$

$\Rightarrow$ enhancement ~~wotu~~ would be 100

$(k = E k_0)$

Mass transfer coefficient $= 100 * 0.01 \frac{\text{cm}}{\text{s}}$

$= 1 \frac{\text{cm}}{\text{s}}$